

DFA (Deterministic Finite Automaton)

Automaton Representation $M=(Q,\Sigma,\delta,q_0,F)$

Q: Set of states $\{q_0,q_1,q_2\}$

Σ : input alphabet $\{a,b\}$ & $\epsilon \notin \Sigma$

δ : transition function $\delta(q,x)=q'$

q_0 : initial state

F: set of accepting states $\{q_2\}$

Language of Automaton $L(M)=\{w \in \Sigma^* : \delta^*(q_0,w) \in F\}$

Languages are regular if a DFA exists for that language.

DFA: Each state has one transition for every alphabet

NFA (Non-deterministic Finite Automaton)

Formal Definition $M=(Q,\Sigma,\Delta,S,F)$

Q: Set of states $\{q_0,q_1,q_2\}$

Σ : input alphabet $\{a,b\}$

Δ : transition function $\Delta(q,x)=\{q_1,q_2,\dots\}$ (include ϵ)

S: initial states $\{q_0\}$

F: set of accepting states $\{q_2\}$

Language of Automaton $L(M) = \{w_1,w_2,\dots,w_n\}$

NFA: Each state can have different transition with the same language output

NFA to DFA Conversion

1. Set initial state of NFA to initial state of DFA

2. Take the states in the DFA, and compute in the NFA what the union Δ of those states for each letter in the alphabet and add them as new states in the DFA.

For example if (q_0,a) takes you to $\{q_1,q_2\}$ add a state $\{q_1,q_2\}$. If there isn't one, the add state null

NFA to DFA Conversion (cont)

3. Set every DFA state as accepting if it contains an accepting state from the NFA

The language for the NFA (M) and DFA (M') are equivalent
 $L(M)=L(M')$

Properties of Regex

$L_1 \cup L_2$ Initial state has two ϵ transitions, one to L_1 and one to L_2

$L_1 L_2$ L_1 accept state transitions (ϵ) to L_2 initial state

L_1^* New initial state transitions to L_1 initial state. New accept state transitions (ϵ) from L_1 accept state. Initial to accept state transition transitions (ϵ) and vice versa.

L_1^R (reverse) Reverse all transitions. Make initial state accepting state and vice versa.

$!(L_1)$ Complement Accepting states become non-accepting and vice versa

$L_1 \cap L_2$ $!(L_1) \cup !(L_2)$

P.S. To turn multiple states to one accept state in an NFA, just add a new accept state, and add transition to the old accept states with language ϵ .

Intersection DFA1 $\cap$ DFA2

Transitions DFA M: $(q_1,p_1) \xrightarrow{a} (q_2,p_2)$
 $M^1: q_1 \xrightarrow{a} q_2$
 $M^2: p_1 \xrightarrow{a} p_2$

Initial State DFA M: (q_0,p_0)
 $M^1: q_0$
 $M^2: p_0$

Accept State DFA M: $(q_i,p_j), (q_i,p_k)$
 $M^1: q_i$
 $M^2: p_j,p_k$

Regular Expressions

$L(r_1+r_2) = L(r_1) \cup L(r_2)$

$L(r_1 \cdot r_2) = L(r_1)L(r_2)$

$L(r_1^*) = (L(r_1))^*$

$L(a) = \{a\}$

Precedence: $*$ $\rightarrow$ $+$ $\rightarrow$ $\cdot$ $\rightarrow$ $\cup$

NFA to Regular Language

1. Transform each transition into regex (e.g. (a,b) is $a+b$)

2. Remove each state one by one, until you are left with the initial and accepting state

3. Resulting regular expression: $r = r_1^* r_2(r_4+r_3r_1^*r_2)^*$ where:

r_1 : initial $\rightarrow$ initial

r_2 : initial $\rightarrow$ accepting

r_3 : accepting $\rightarrow$ initial

r_4 : accepting $\rightarrow$ accepting

Proving Regularity with Pumping Lemma

Prove that an infinite language L is not regular:

1. Assume L is regular

2. The pumping lemma should hold for L

3. Use the pumping lemma to obtain a contradiction:

a. Let m be the critical length for L

b. Choose a particular string $w \in L$ which satisfies the length condition $|w| \geq m$

c. Write $w=xyz$

d. Show that $w^i = xy^i z \notin L$ for some $i \neq 1$