

Quadratic Equation

Roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\frac{b}{a}$;

Product of roots $x_1 x_2 = \frac{c}{a}$

For real roots, $b^2 - 4ac \geq 0$
For imaginary roots, $b^2 - 4ac < 0$

Logarithm

$\log_{10} N = x \Rightarrow 10^x = N$
 $\log_b N = \log_a N \cdot \log_a b$
 $\log_b 1 = 0, \log_a a = 1$

$\log mn = \log m + \log n$ $\log \frac{m}{n} = \log m - \log n$

$\log m^n = n \log m$ $\log m = 2.303 \log_{10} m$
 $\log 2 = 0.3010$ $\log 3 = 0.4771$

Arithmetic Progression

$a, a+d, a+2d, a+3d, \dots, a+(n-1)d$
here d = common difference

Sum of n terms $S_n = \frac{n}{2} [2a + (n-1)d]$

n^{th} term, $a_n = a + (n-1)d$

Geometric Progression

$a, ar, ar^2, ar^3, \dots$ here, r = common ratio

n^{th} term, $a_n = a \cdot r^{n-1}$

Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$

Sum of ∞ terms $S_\infty = \frac{a}{1-r}$ [where $|r| < 1$]

Cosine Rule

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Binomial Theorem

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \frac{n(n-1)(n-2)}{6} x^3 + \dots$

$(1-x)^n = 1 - nx + \frac{n(n-1)}{2} x^2 - \frac{n(n-1)(n-2)}{6} x^3 + \dots$

If $x \ll 1$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

Trigonometry I

$2\pi \text{ radian} = 360^\circ \Rightarrow 1 \text{ rad} = 57.3^\circ$

$\sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$ $\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{perpendicular}}{\text{base}}$ $\sec \theta = \frac{\text{hypotenuse}}{\text{base}}$ $\csc \theta = \frac{\text{hypotenuse}}{\text{perpendicular}}$ $\cot \theta = \frac{\text{base}}{\text{perpendicular}}$

$\sin \theta = \frac{a}{\sqrt{a^2 + b^2}}$ $\cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$ $\tan \theta = \frac{a}{b}$ $\cot \theta = \frac{1}{\tan \theta}$

$\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$

Trigonometry II

$\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ $\sin 3A = 3 \sin A - 4 \sin^3 A$

$\cos 3A = 4 \cos^3 A - 3 \cos A$ $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$2 \cos A \cos B = \cos(A-B) + \cos(A+B)$ $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

Trigonometry III

$\frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\cos \theta} = \frac{\tan \theta}{\tan \theta} = \frac{\sec \theta}{\sec \theta} = \frac{\csc \theta}{\csc \theta} = \frac{\cot \theta}{\cot \theta} = 1$

$\frac{\sin \theta}{\sin \theta} = \frac{\cos \theta}{\cos \theta} = \frac{\tan \theta}{\tan \theta} = \frac{\sec \theta}{\sec \theta} = \frac{\csc \theta}{\csc \theta} = \frac{\cot \theta}{\cot \theta} = 1$

Trigonometry IV

θ	0° (0)	30° ($\pi/6$)	45° ($\pi/4$)	60° ($\pi/3$)	90° ($\pi/2$)	120° ($2\pi/3$)	135° ($3\pi/4$)	150° ($5\pi/6$)	180° (π)	270° ($3\pi/2$)	360° (2π)
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	∞	0

Trigonometry V

$\sin(90^\circ - \theta) = \cos \theta$ $\sin(180^\circ - \theta) = \sin \theta$ $\sin(-\theta) = -\sin \theta$ $\sin(90^\circ + \theta) = \cos \theta$

$\cos(90^\circ + \theta) = -\sin \theta$ $\cos(180^\circ - \theta) = -\cos \theta$ $\cos(-\theta) = \cos \theta$ $\cos(90^\circ - \theta) = \sin \theta$

$\tan(90^\circ + \theta) = -\cot \theta$ $\tan(180^\circ - \theta) = -\tan \theta$ $\tan(-\theta) = -\tan \theta$ $\tan(90^\circ - \theta) = \cot \theta$

$\sin(180^\circ + \theta) = -\sin \theta$ $\sin(270^\circ - \theta) = -\cos \theta$ $\sin(270^\circ + \theta) = -\cos \theta$ $\sin(360^\circ - \theta) = -\sin \theta$

$\cos(180^\circ + \theta) = -\cos \theta$ $\cos(270^\circ - \theta) = \sin \theta$ $\cos(270^\circ + \theta) = \sin \theta$ $\cos(360^\circ - \theta) = \cos \theta$

$\tan(180^\circ + \theta) = \tan \theta$ $\tan(270^\circ - \theta) = \cot \theta$ $\tan(270^\circ + \theta) = -\cot \theta$ $\tan(360^\circ - \theta) = -\tan \theta$

For Small Theta

$\sin \theta \approx \theta$ $\cos \theta \approx 1$ $\tan \theta \approx \theta$ $\sin \theta \approx \tan \theta$

Average of a Varying Quantity

If $y = f(x)$ then $\langle y \rangle = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} y dx$

Integration

C = Arbitrary constant, k = constant

- $\int f(x) dx = g(x) + C$
- $\frac{d}{dx} (g(x)) = f(x)$
- $\int k f(x) dx = k \int f(x) dx$
- $\int (u + v + w) dx = \int u dx + \int v dx + \int w dx$
- $\int e^x dx = e^x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x} dx = \ln x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$
- $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C$

Differentiation

- $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$ $y = f(x) \rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \rightarrow \frac{dy}{dx} = \cos x$ $y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$
- $y = e^{ax+b} \rightarrow \frac{dy}{dx} = ae^{ax+b}$ $y = uv \rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
- $y = f(g(x)) \rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \cdot \frac{dg(x)}{dx}$
- $y = k(\text{constant}) \rightarrow \frac{dy}{dx} = 0$
- $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Maxima and Minima

For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} < -ve$
For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} > +ve$

Componendo & Dividendo Theorem

If $\frac{p}{q} = \frac{a}{b}$ then $\frac{p+q}{p-q} = \frac{a+b}{a-b}$