

Definitions

Sample Space	The set of all possible outcomes of an experiment is called the sample space and is denoted by Ω .		
Sigma field	A collection of sets F of Ω is called a σ -field if it satisfies the following conditions:		
	1. $\emptyset \in F$	2. If $A_1, \dots, A_n \in F$ then $\bigcup_{i=1}^n A_i \in F$	3. If $A \in F$ then $A^c \in F$
Probability	A probability measure P on (Ω, F) is a function $P : F \rightarrow [0, 1]$ which satisfies:		
	1. $P(\Omega)=1$ and $P(\emptyset)=0$	2.	
Conditional Probability	Consider probability space (Ω, F, P) and let $A, B \in F$ with $P(B) > 0$. Then the conditional probability that A occurs given B occurs is defined to be: $P(A B) = P(A \cap B) / P(B)$		
Total Probability	A family of sets $B_1, \dots, B_n$ is called a partition of Ω if: $\forall i \neq j, B_i \cap B_j = \emptyset$ and $\bigcup_{i=1}^n B_i = \Omega$	$P(A) = \sum_{i=1}^n P(A \cap B_i)$	$P(A) = \sum_{i=1}^n P(A B_i)P(B_i)$
Independence	Consider probability space (Ω, F, P) and let $A, B \in F$. A and B are independent if $P(A \cap B) = P(A)P(B)$		
	More generally, a family of F -sets $A_1, \dots, A_n$ ($n \geq 2$) are independent if $P(\bigcap_{i=1}^n A_i) = \prod_{i=1}^n P(A_i)$		



By madsonic

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Definitions (cont)

Random Variable (RV)	A RV is a function $X : \Omega \rightarrow \mathbb{R}$ such that for each $x \in \mathbb{R}$, $\{\omega \in \Omega : X(\omega) \leq x\} \in \mathcal{F}$. Such a function is said to be $\mathcal{F}$ -measurable	
Distribution Function	Distribution function of a random variable X is the function $F : \mathbb{R} \rightarrow [0, 1]$ given by $F(x) = P(X \leq x)$, $x \in \mathbb{R}$.	
Discrete RV	A RV is said to be discrete if it takes values in some countable subset $X = \{x_1, x_2, \dots\}$ of $\mathbb{R}$	
PMF	PMF of a discrete RV X , is the function $f : X \rightarrow [0, 1]$ defined by $f(x) = P(X = x)$. It satisfy:	PDF function f is called the probability density function (PDF) of the continuous random variable X
	1. set of x s.t. $f(x) \neq 0$ is countable	$f(x) = F'(x)$
	2. $\sum_{x \in X} f(x) = 1$	$F(x) = \int_{-\infty}^x f(u) du$
	3. $f(x) \geq 0$	
Independence	Discrete RV X and Y are indie if the events $\{X = x\}$ & $\{Y = y\}$ are indie for each $(x, y) \in X \times Y$	The RV X and Y are indie if $\{X \leq x\}$ $\{Y \leq y\}$ are indie events for each $x, y \in \mathbb{R}$
	$P(X, Y) = P(X=x)P(Y=y)$	
	$f(x, y) = f(x)f(y)$	$f(x, y) = f(x)f(y) \quad F(x, y) \quad v$
	$E[XY] = E[X]E[Y]$	
Expectation	expected value of RV X on X ,	The expectation of a continuous random variable X with PDF f is given by
	$E[X] = \sum_{x \in X} xf(x)$	$E[X] = \int_{-\infty}^{\infty} xf(x) dx$
	$E[g(x)] = \sum_{x \in X} g(x)f(x)$	$E[g(x)] = \int_{-\infty}^{\infty} g(x)f(x) dx$
Variance	spread of RV	$E[(X - E[X])^2] = E[X^2] - E[X]^2$
MGF (uniquely characterises distribution)	$M(t) = E[e^{Xt}] = \sum_{x \in X} e^{xt} f(x)$	$t \in T$ s.t. t for $\sum_{x \in X} e^{xt} f(x) < \infty$
	$M(t) = E[e^{Xt}] = \int_{-\infty}^{\infty} e^{xt} f(x) dx$	$t \in T$ s.t. t for $\int_{-\infty}^{\infty} e^{xt} f(x) dx < \infty$



Definitions (cont)		
	$M(t_1, t_2) = E[e^{Xt_1 + Yt_2}] = \int \int e^{Xt_1 + Yt_2} f(x, y) dx dy \quad (t_1, t_2) \in T$	$E[X] = \partial/\partial t_1 M(t_1, t_2) \quad E[XY] = \partial^2/\partial t_1 \partial t_2 M(t_1, t_2) \mid_{t_1=t_2=0}$
	$E[X^k] = M^k(0)$	
Moment	Given a discrete RV X on X , with PMF f and $k \in \mathbb{Z}^+$, the k^{th} moment of X is	$E[X^k]$
Central Moment	k^{th} central moment of X is	$E[(X - E[X])^k]$
Dependence	Joint distribution function $F : \mathbb{R}^2 \rightarrow [0, 1]$ of X, Y where X and Y are discrete random variables, is given by $F(x, y) = P(X \leq x \text{ and } Y \leq y)$	The joint distribution function of X and Y is the function $F : \mathbb{R}^2 \rightarrow [0, 1]$ given by $F(x, y) = P(X \leq x, Y \leq y)$
	Joint mass function $f : \mathbb{R}^2 \rightarrow [0, 1]$ is given by $f(x, y) = P(X=x, Y=y)$	The random variables are jointly continuous with joint PDF $f : \mathbb{R}^2 \rightarrow [0, \infty)$ if $F(x, y) = \int_{-\infty}^y \int_{-\infty}^x f(u, v) du dv$
		$f(x, y) = \partial^2/\partial x \partial y F(x, y)$
Marginal	$f(x) = \sum_{y \in Y} f(x, y)$	$f(x) = \int_{y \in Y} f(x, y) dy \quad F(x) = \lim_{y \rightarrow \infty} F(x, y) \quad F(x) = \int_{-\infty}^x \int_{-\infty}^{\infty} f(u, y) dy du$
	$E[g(x, y)] = \sum_{x, y \in X \times Y} g(x, y) f(x, y)$	$E[g(x, y)] = \int_{x, y \in X \times Y} g(x, y) f(x, y) dx dy$
Covariance	indie $\Rightarrow E[XY] = E[X]E[Y], \text{Cov} = 0 \Rightarrow \rho = 0$	$\rho = 0 \Rightarrow E[XY] = E[X]E[Y]$
	$\text{Cov}[X, Y] = E[(X - E[X])(Y - E[Y])]$	$\text{Cov}[X, Y] = E[XY] - E[X]E[Y]$
Correlation	Gives linear relationship (+/-). $ \rho $ close to 1 is strong, close to 0 is weak	special for bi-variate normal, indie $\Leftrightarrow$ uncorrelated
	$\rho(X, Y) = \text{Cov}[X, Y] / \sqrt{\text{Var}[X]\text{Var}[Y]}$	
Conditional distribution	The conditional distribution function of Y given X , written $F_{Y X}(\cdot x)$, is defined by	$F(y x) = \int_{-\infty}^y f(x, v)/f(x) dv \quad f(y x) = f(x, y)/f(x) \text{ where } f(x) = \int_{-\infty}^{\infty} f(x, y) dy$



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Definitions (cont)

$$F_{Y|X}(y|x) = P(Y \leq y | X = x)$$

for any x with $P(X = x) > 0$. The conditional PMF of Y given $X = x$ is defined by ... when x is s.t. $P(X = x) > 0$

$$f(y|x) = P(Y = y | X = x)$$

$$f(x, y) = f(x|y)f(y) \text{ or } f(y|x)f(x)$$

Conditional expectation

The conditional expectation of a RV Y , given $X = x$ is $E[Y|X = x] = \sum_{y \in Y} y f(y|x)$ given that the conditional PMF is well-defined

$$E[h(X)g(Y)] = E[E[g(Y)|X]h(X)] = \int (\int g(Y)f(Y|X) dx) h(X)f(x) dx$$

$$E[Y|X = x] = \sum_{y \in Y} y f(y|x)$$

$$E[E[Y|X]] = E[Y] \quad E[E[Y|X]g(X)] = E[Yg(X)]$$

$$E[aX + bY|Z] = aE[X|Z] + bE[Y|Z]$$

if X and Y are independent

$$E[X|Y] = E[X] \quad \text{Var}[X|Y] = E[X^2|Y] - E[X|Y]^2$$

Theorems

Bayes Theorem

Consider probability space $(\Omega, \mathcal{F}, P)$ and let $A, B \in \mathcal{F}$ with $P(A), P(B) > 0$. Then we have:

$$P(B|A) = P(A|B)P(B) / P(A)$$

Independence

If X and Y are indie RV and $g : X \rightarrow \mathbb{R}, h : Y \rightarrow \mathbb{R}$, then the RV $g(X)$ and $h(Y)$ are also indie

Expectations

1. if $X \geq 0$, $E[X] \geq 0$

2. if $a, b \in \mathbb{R}$ then $E[aX + bY] = aE[X] + bE[Y]$

3. if $X = c \in \mathbb{R}$ always, then $E[X] = c$.

Variance

1. For $a \in \mathbb{R}$, $\text{Var}[aX] = a^2 \text{Var}[X]$

2. Uncorrelated $\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y]$

Conditional Expectation

Conditional expectations satisfies $E[E[Y|X]] = E[Y]$ assuming all the expectations exist

for any $g : \mathbb{R} \rightarrow \mathbb{R}$, $E[E[Y|X]g(X)] = E[Yg(X)]$ assuming all expectations exist

Change of variable

If (X_1, X_2) have joint density $f(x, y)$ on Z , then for $(Y_1, Y_2) = T(X_1, X_2)$, with T as described above, the joint density of (Y_1, Y_2) , denoted g is: $g(y_1, y_2) = f(T^{-1}(y_1, y_2), T^{-1}(y_1, y_2)) |J(y_1, y_2)|$ $(y_1, y_2) \in T$



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