

Solving Problems Using Linear Equations

Worked Example 6:

Mira is twice Sanjin's age. How old are Mira and Sanjin now if in 10 years the sum of their ages will be 89. Use s to represent Sanjin's age.

Sanjin's Age: s Mira's present age: $2s$

In 10 years:

Sanjin = $s + 10$ Mira's age = $2s + 10$

$$(s+10) + (2s+10) = 89$$

$$3s + 20 = 89$$

$$3s = 69$$

$$s = 23$$

$$2s = 2 \times 23 = 46$$

Sanjin is 23 years old and Mira is 46 years old. In 10 years they will be 33 and 56 old respectively; $33 + 56 = 89$

Distance and Midpoint

Distance Formula	Midpoint Formula
The distance, d , between any two Cartesian coordinates (x_1, y_1) and (x_2, y_2) is:	The midpoint $M(x,y)$ between 2 points (x_1, y_1) and (x_2, y_2) is given by:
$d =$	$M(x,y) =$

Gradient

The gradient of a line can be found by the ratio of the vertical **rise** to the horizontal **run** between any two points on the line.

Gradient $AB = (\text{vertical rise})/(\text{horizontal run})$

The letter m is used to represent the gradient. $y=mx+b$

$$m = (\text{rise})/(\text{run})$$

$$= (\text{change in } y\text{-value})/(\text{change in } x\text{-value})$$

$$= (y_2 - y_1)/(x_2 - x_1)$$

Linear Graphs: Gradient and Y-Intercept

The **gradient-intercept form**:

$y=mx+b$, where m is the gradient and b is the y-intercept.

The **general form**:

$ax+by+c=0$, where a , b and c are constants and $a \neq 0$, $b \neq 0$.

Linear Graphs: Gradient and Y-Intercept (cont)

In this equation, the gradient $m = (-a)/(b)$ and the y-intercept $= (-c)/(b)$ can be found by rearranging the equation to make y the subject.

Worked Example 15

$$(a) y = -3x+2$$

$$\text{gradient} = -3$$

$$\text{y-intercept: } (0,2)$$

$$(b) y = x-1$$

$$= 1x+(-1)$$

$$\text{gradient} = 1$$

$$\text{y-intercept: } (0,-1)$$

$$(c) 2y = 5x - 4$$

$$y = (5)/(2)x - 2$$

$$\text{gradient} = (5)/(2)$$

$$\text{y-intercept: } (0,-2)$$

$$(d) y+2x = 6$$

$$y = 6-2x$$

$$y = -2x+6$$

$$\text{gradient: } -2$$

$$\text{y-intercept: } (0,6)$$

Sketching Linear Graphs

Graphing using the y-intercept and gradient

There is no need to plot a number of points to graph linear relationships. Because only 2 points are needed to define a line, use the y-intercept as one point on the graph and the gradient, m to locate the second point. Once 2 points are known, a straight line can be drawn through them.

Sketching other Linear Graphs, $y=mx+b$, $b \neq 0$

If the y-intercept is not the origin, then first identify the y-intercept from the equation before using the gradient to find the second point.

Using the equation $y=mx+b$ and substituting $x = 0$, you get $y = b$.

$x = 0$ is the x-coordinate of a point on the y-axis, so b is the y-value at the point at which the graph crosses the y-axis.

This is the y-intercept

Worked Example 17

Sketching Linear Graphs (cont)

Use the y-intercept and the gradient to sketch the graph of $y = 2x+1$

y-intercept: $(0,1)$

Gradient = 2

$$= (2)/(1) = (\text{rise})/(\text{run})$$

Parallel and Perpendicular Lines

Worked Example 19

Find the gradient of the line parallel to each of the following

$$(a) y = 2x-5$$

$$m = 2$$

Any parallel line will have a gradient of 2.

$$(b) 2x + 3y = 6$$

$$2x - 2x + 3y = 6 - 2x$$

$$(3y)/(3) = - (2)/(3)x + (6)/(3)$$

$$y = - (2)/(3)x + 2$$

$$m_1 = - (2)/(3)$$

Any parallel line will have a gradient of $- (2)/(3)$.

Worked Example 20

Find the gradient of a line perpendicular to each of the following

$$(a) y = 4x + 3$$

$$m_1 = 4$$

$$m_2 = - (1)/(m_1)$$

$$= - (1)/(4)$$

$$(b) 3x-5y = 10$$

$$3x - 3x - 5y = 10 - 3x$$

$$-5y = -3x + 10$$

$$(-5y)/(-5) = (-3x)/(-5) + (10)/(-5)$$

$$y = (3)/(5)x - 2 \quad m_1 = (3)/(5)$$

$$m_2 = - (1)/(m_1) = -(5)/(3)$$

