

Notation						
Name	Operation	y versions	f(x) versions	Composition versions	Second derivative	nth derivative
Leibniz/Fraction Notation	$d/dx (f(x))=d/dx (y)$	$dy/dx=dy(x)/dx$	$df/dx=df(x)/dx=d(f(x))/dx$	$df/dg*dg/dx$	d^2f/dx^2	$d^n/dx^n=d^n f/dx^n$
Lagrange/Prime Notation	$d/dx (f(x))=d/dx (y)$	y'	$f'=f'(x)=(f(x))'$	$(f(g(x)))'$	f''	$f^{(n)}(x)$
Newton/Dot Notation	$d/dt (f(t))=d/dt (y(t))$	$\dot{y}$			$\ddot{y}$	
Euler/D-Notation	$D_x(f)$	$D_x y$	$D_x f$	$D_x(f(g))$	$D_x^2 f$	$D_x^n f$
$n \in \mathbb{N}_1 = \{1,2,3,4,5,\dots\}$						

Derivative Rules			
Formal/Limit Definition of a Derivative	$f'(x)=\lim_{h \rightarrow 0} (f(x+h)-f(x))/h$		
Limit Definition of the Derivative at a point	$f'(a)=\lim_{h \rightarrow 0} (f(a+h)-f(a))/h$	$f'(a)=\lim_{x \rightarrow a} (f(x)-f(a))/(x-a)$	
Linearity 1: Constant-Multiple Rule	$d/dx (kf(x))$	$k*d/dx (f)$	kf'
Linearity 2: Sum/Difference Rule	$d/dx (f(x)\pm g(x))$	$d/dx (f) \pm d/dx (g)$	$f'\pm g'$
Product Rule	$d/dx (f(x)*g(x))$	$f'g+fg'$	
Multi-Product Rule	$d/dx (p(x)*q(x)*r(x)*s(x)*\dots)$	$p'qrs\dots + pq'rs\dots + pqr's\dots + pqr s' \dots + \dots$	$pqr\dots*(p'/p + q'/q + r'/r + s'/s + \dots)$
Quotient Rule	$d/dx (f(x)/g(x))$	$(f'g-fg')/g^2$	$g(x)\neq 0$, quotients can be rewritten into products with sign-flipped exponents
Chain Rule	$d/dx (f(g(x)))$	$f'(g)g'$	
Multi-Chain Rule	$d/dx (p(q(r(s(\dots))))))$	$p'(q(r(s(\dots))))*q'(r(s(\dots)))*r'(s(\dots))*s'(\dots)*\dots$	
First Fundamental Theorem of Calculus (FTC I)	$d/dx (\int_a^x f(t)dt)$	$f(x)$	Derivatives and integrals are inverses of each other
FTC I Chain Rule 1	$d/dx (\int_a^{v(x)} f(t)dt)$	$f(v)v'$	
FTC I Chain Rule 2	$d/dx (\int_{u(x)}^{v(x)} f(t)dt)$	$f(v)v'-f(u)u'$	
Summation Rule	$d/dx (\sum f(x))$	$\sum f'(x)$	The summation must be within its interval of convergence

a and k are constants

f, g, p, q, r, s, u, and v are functions of x such that $f=f(x)$, $g=g(x)$, $p=p(x)$, $q=q(x)$, $r=r(x)$, $s=s(x)$, $u=u(x)$, and $v=v(x)$, unless otherwise shown

Derivatives of Algebraic Functions						
Rule	Function	Derivative	nth Derivative of Function	nth Derivative	Function Composition	Derivative by Chain Rule



Derivatives of Algebraic Functions (cont)

Constant	$d/dx (k)$	0	$d^n/dx^n (k)$	0	$d/dx (f(k))$	0
Power	$d/dx (x^k)$	kx^{k-1}	$d^n/dx^n (x^k), k \neq 0, k-n+1 \neq 0$	$\Gamma(k+1)x^{k-n}/\Gamma(k-n+1)$	$d/dx (u(x)^k), u(x) \neq 0$	$ku^{k-1}u'$
Natural Exponential	$d/dx (e^x)$	e^x	$d^n/dx^n (e^x)$	e^x	$d/dx (e^{u(x)})$	$e^u u'$
Natural Logarithm	$d/dx (\ln(x))$	$1/x$	$d^n/dx^n (\ln(x))$	$(-1)^{n+1}(n-1)!/x^n$	$d/dx (\ln(u(x))), u(x) > 0$	u'/u
General Exponential	$d/dx (k^x), k > 0$	$k^x \ln(k)$	$d^n/dx^n (k^x), k > 0$	$k^x (\ln(k))^n$	$d/dx (k^{u(x)}), k > 0$	$k^u \ln(k) u'$
General Logarithm	$d/dx (\log_k(x)), k > 0, k \neq 1$	$1/(x \ln(k))$	$d^n/dx^n (\log_k(x)), k > 0, k \neq 1$	$(-1)^{n+1}(n-1)!/(x^n \ln(k))$	$d/dx (\log_k(u(x))), k > 0, k \neq 1, u(x) \neq 0$	$u'/(u \ln(k))$
Absolute Value	$d/dx (x), x \neq 0$	$x/ x $ OR $ x /x$			$d/dx (u(x)), u(x) \neq 0$	$u^* u'/ u $ OR $ u ^* u'/u$
Function-Power--Function	$d/dx (f(x)^{g(x)}), f(x) > 0$	$f^g (f^g g' + \ln(f) g')$				

k is a constant

f=f(x), g=g(x), and u=u(x) are all functions of the variable x

$m, n \in \mathbb{N}_1 = \{1, 2, 3, 4, 5, \dots\}$

$\Gamma(x)$ is the gamma function, which defines factorials for negative/non-integer numbers

$x! = \Gamma(x+1)$

$n! = n(n-1)! = n(n-1)(n-2)! = n(n-1)(n-2)(n-3)! = \dots$

$n! = n(n-1)(n-2)(n-3) \dots \cdot 3 \cdot 2 \cdot 1$

$0! = 1, 1! = 1$

Derivatives of Trigonometric Functions

Standard Trigonometric	Derivative	Inverse Trigonometric	Derivative	Hyperbolic Trigonometric	Derivative	Hyperbolic Inverse Trigonometric	Derivative
$d/dx (\sin(x))$	$\cos(x)$	$d/dx (\arcsin(x))$	$1/\sqrt{1-x^2}$	$d/dx (\sinh(x))$	$\cosh(x)$	$d/dx (\operatorname{arcsinh}(x))$	$1/\sqrt{1+x^2}$
$d/dx (\cos(x))$	$-\sin(x)$	$d/dx (\arccos(x))$	$-1/\sqrt{1-x^2}$	$d/dx (\cosh(x))$	$\sinh(x)$	$d/dx (\operatorname{arccosh}(x))$	$-1/\sqrt{x^2-1}$
$d/dx (\tan(x))$	$\sec^2(x)$	$d/dx (\arctan(x))$	$1/(1+x^2)$	$d/dx (\tanh(x))$	$\operatorname{sech}^2(x)$	$d/dx (\operatorname{arctanh}(x))$	$1/(1-x^2)$
$d/dx (\csc(x))$	$-\csc(x)\cot(x)$	$d/dx (\operatorname{arccsc}(x))$	$-1/(x \sqrt{x^2-1})$	$d/dx (\operatorname{csch}(x))$	$-\operatorname{csch}(x)\coth(x)$	$d/dx (\operatorname{arccsch}(x))$	$-1/(x \sqrt{x^2+1})$
$d/dx (\sec(x))$	$\sec(x)\tan(x)$	$d/dx (\operatorname{arcsec}(x))$	$1/(x \sqrt{x^2-1})$	$d/dx (\operatorname{sech}(x))$	$-\operatorname{sech}(x)\tanh(x)$	$d/dx (\operatorname{arcsech}(x))$	$-1/(x \sqrt{1-x^2})$
$d/dx (\cot(x))$	$-\csc^2(x)$	$d/dx (\operatorname{arccot}(x))$	$-1/(1+x^2)$	$d/dx (\coth(x))$	$-\operatorname{csch}^2(x)$	$d/dx (\operatorname{arccoth}(x))$	$1/(1-x^2)$

$d^n/dx^n (\sin(x)) = \sin(x+n\pi/2)$

$d^n/dx^n (\cos(x)) = \cos(x+n\pi/2)$

$\sinh(x) = (e^x - e^{-x})/2$

$\cosh(x) = (e^x + e^{-x})/2$

$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2+1})$

$\operatorname{arccosh}(x) = \ln(x + \sqrt{x^2-1}), x \geq 1$

Polynomial Derivative Examples

Trigonometric Derivative Examples (cont)

$$d/dx (\sin(\arccos(x))) = -x/\sqrt{1-x^2}$$

$$d/dx (\sin(k)) = 0$$

Exponential Derivative Examples

$d/dx (x)$	1
$d/dx (x^2)$	$2x$
$d/dx (x^3)$	$3x^2$
$d/dx (x^4)$	$4x^3$
$d/dx (1/x)$	$-1/x^2$
$d/dx (-1/x^2)$	$2/x^3$
$d/dx (2/x^3)$	$-6/x^4$
$d/dx (-6/x^4)$	$24/x^5$
$d/dx (\sqrt{x})$	$1/(2\sqrt{x})$
$d/dx (x^{1/3})$	$1/(3x^{2/3})$
$d/dx (x^{1/4})$	$1/(4x^{3/4})$
$d/dx (x^{3/2})$	$3\sqrt{x}/2$
$d/dx (x^{5/3})$	$5x^{2/3}/3$
$d/dx (x^{-\sqrt{2}-3})$	$(-\sqrt{2}-3)x^{-\sqrt{2}-4}$
$d/dx (1/(1+x))$	$-1/(1+x)^2$
$d/dx (-1/(1+x)^2)$	$2/(1+x)^3$
$d/dx (-1/(1-x))$	$-1/(1-x)^2$
$d/dx (-1/(1-x)^2)$	$-2/(1-x)^3$
$d/dx (\sqrt{5x+1})$	$5/(2\sqrt{4x+1})$
$d/dx (\sqrt{x^5+1})$	$5x^4/(2\sqrt{x^5+1})$
$d/dx ((2x^2+5)^9)$	$36x(2x^2+5)^8$
$d/dx (1)$	0

Special/Other Derivative Examples	
$d/dx (e^x \sin(x))$	$e^x \sin(x) + e^x \cos(x)$
$d/dx (e^x \cos(x))$	$e^x \cos(x) - e^x \sin(x)$
$d/dx (\sin^x(x))$	$\sin^x(x)(\ln(\sin(x)) + x \cot(x))$
$d/dx (\sin(x)^{\cos(x)})$	$\sin(x)^{\cos(x)}(\cos^2(x) \csc(x) - \sin(x) \ln(\sin(x)))$
$d/dx (\ln(1/(1-x)))$	$1/(1-x)$
$d/dx (\ln(x^3+7x+12))$	$(3x^2+7)/(x^3+7x+12)$
$d/dx (\ln(e^{3x} \tan(x^3)))$	$3 + (3x^2 \sec^2(x^3))/(\tan(x^3))$
$d/dx (1+k+t+\sqrt{2+\cos(a)}+e+\pi+-\ln(3))$	0

Trigonometric Derivative Examples	
$d/dx (-\sin(x))$	$-\cos(x)$
$d/dx (-\cos(x))$	$\sin(x)$
$d/dx (\sin(2x))$	$2\cos(2x)$
$d/dx (\cos(2x))$	$-2\sin(2x)$
$d/dx (\sin^2(x))$	$2\sin(x)\cos(x)$
$d/dx (\cos^2(x))$	$-2\cos(x)\sin(x)$
$d/dx (\arctan(3x))$	$3/(1+9x^2)$
$d/dx (\sin(\sin(x)))$	$\cos(x)\cos(\sin(x))$

$d/dx (xe^x)$	$e^x + xe^x$
$d/dx (e^{2x})$	$2e^{2x}$
$d/dx (e^{x^2})$	$2xe^{x^2}$
$d/dx (e^{e^x})$	$e^x e^{e^x}$
$d/dx (x^x)$	$x^x(\ln(x)+1)$
$d/dx (2^{3^x})$	$2^{3^x} 3^x \ln(2) \ln(3)$
$d/dx (e^k)$	0

Logarithmic Derivative Examples	
$d/dx (\ln(1/x))$	$-1/x$
$d/dx (\ln(1+x))$	$1/(1+x)$
$d/dx (\ln(1-x))$	$-1/(1-x)$
$d/dx (\ln(x^2))$	$2/x$
$d/dx (\ln(x^3))$	$3/x$
$d/dx (\ln(x^4))$	$4/x$
$d/dx (x \ln(x))$	$\ln(x) + 1$
$d/dx (\ln(\ln(x)))$	$1/(x \ln(x))$
$d/dx (\ln(k))$	0

