

Geometric Mean

Best explained as an example, The geometric mean of $a = 9$ and $b = 4$ is 6, because $6 = \sqrt{9 \cdot 4}$.

Geometric Mean Right Triangle Theorems

Theorems Right Triangle Geometric Mean Theorems

8.2 Geometric Mean (Altitude) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{a}{h} = \frac{h}{b}$ or $h = \sqrt{ab}$.

8.3 Geometric Mean (Leg) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{c}{a} = \frac{a}{x}$ or $a = \sqrt{cx}$ and $\frac{c}{b} = \frac{b}{y}$ or $b = \sqrt{cy}$.

EXAMPLE I CANT EXPLAIN

Example 3 Use Two Angles of Elevation or Depression

TREE REMOVAL To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a 70° angle of elevation. She then steps back 10 meters and sights the top at a 26° angle. If Mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?

Understand $\triangle ABC$ and $\triangle ABD$ are right triangles. The height of the tree is the sum of Mrs. Long's height and AB .

Plan Since her initial distance from the tree is not given, write and solve a system of equations using both triangles. Let $AB = x$ and $CB = y$. So $DB = y + 10$ and the height of the tree is $x + 1.7$.

Continued

Solve Use $\triangle ABC$.

$$\tan 70^\circ = \frac{x}{y} \quad \tan = \frac{\text{opposite}}{\text{adjacent}} \quad m\angle ACB = 70^\circ$$

$$y \tan 70^\circ = x \quad \text{Multiply each side by } y.$$

Use $\triangle ABD$.

$$\tan 26^\circ = \frac{x}{y + 10} \quad \tan = \frac{\text{opposite}}{\text{adjacent}} \quad m\angle D = 26^\circ$$

$$(y + 10) \tan 26^\circ = x \quad \text{Multiply each side by } y + 10.$$

Substitute the value for x from $\triangle ABD$ in the equation for $\triangle ABC$ and solve for y .

$$y \tan 70^\circ = x$$

$$y \tan 70^\circ = (y + 10) \tan 26^\circ$$

$$y \tan 70^\circ = y \tan 26^\circ + 10 \tan 26^\circ$$

$$y \tan 70^\circ - y \tan 26^\circ = 10 \tan 26^\circ$$

$$y(\tan 70^\circ - \tan 26^\circ) = 10 \tan 26^\circ$$

$$y = \frac{10 \tan 26^\circ}{\tan 70^\circ - \tan 26^\circ}$$

Use a calculator to find that $y \approx 2.16$. Using the equation from $\triangle ABC$, $x = 2.16 \tan 70^\circ$ or about 5.9.

The height of the tree is $5.9 + 1.7$ or 7.6, which is about 8 meters.

Check Substitute the value for y in the equation from $\triangle ABD$.

$$x = (2.16 + 10) \tan 26^\circ \text{ or about } 5.9. \text{ This is the same value found using the equation from } \triangle ABC. \checkmark$$

Pythagorean Theorem

$a^2 + b^2 = c^2$ (Only for Right Triangles)

30-90-60 Triangles and 45-90-45 Triangles

REVIEW: Special Right Triangles

45-45-90 isosceles right triangle

Leg x , Leg x , Hypotenuse $x\sqrt{2}$

30-60-90 special right triangle

Short Leg x , Long Leg $x\sqrt{3}$, Hypotenuse $2x$

Soh Cah Toa

Sin()	Opposite / Hypotenuse
Cos()	Adjacent / Hypotenuse
Tan()	Opposite / Adjacent

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measures A , B , and C , respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A \quad b^2 = a^2 + c^2 - 2ac \cos B \quad c^2 = a^2 + b^2 - 2ab \cos C$$